Binding of Quarks and the πN σ-Term.

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Abstract

It is shown that the binding effect that is associated with the short range part of the Goldstone boson exchange interaction between constituent quarks provides a good description of the πN σ-term.

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The pion-nucleon σ-term [1]

$$\sigma_{\pi N} = \frac{1}{2}(m_u^0 + m_d^0) < N|\bar{u}u + \bar{d}d|N >, \tag{1}$$

is a measure of the explicit chiral symmetry breaking effects in the nucleon. Here m_u^0 and m_d^0 stand for the current quark masses. Its experimental value may be extracted from pion-nucleon scattering data, the most recent result being [2]

$$\sigma_{\pi N} = 45 \pm 10 \ MeV. \tag{2}$$

Clearly that any successful model of the nucleon should be able to explain this empirical value. The additive quark ansatz within naive constituent quark model as well as in the extended Nambu-Jona-Lasinio model [3, 4] leads to a much smaller value for $\sigma_{\pi N}$. This indicates that some essential piece of physics is absent within the additive quark ansatz. The aim of this letter is to show that the effects responsible for the binding of the quarks in the nucleon are of crucial importance for the explanation of the emperical value of $\sigma_{\pi N}$.

The view that in the low-energy regime, i.e. beyond the spontaneous breaking of chiral symmetry, light and strange baryons can be viewed as systems of three constituent quarks which interact by the exchange of Goldstone bosons (pseudoscalar mesons) and which are subject to confinement [5] is becoming rather compelling [6]. Such an interaction between the light quarks in heavy baryons containing one heavy quark is important for understanding the spectra of the heavy flavor hyperons as well [7]. The creation of the quark-antiquark sea in the nonperturbative regime via the coupling of the Goldstone bosons to the valence quarks also resolves some well-known problems related to the spin and flavor content of the nucleon that appear in naive constituent quark and parton models [8, 9, 10]. Below we show that the contribution to $\sigma_{\pi N}$ that arises from the short-range part of Goldstone boson exchange (GBE) between the constituent quarks is crucial for the explanation of its empirical value.

The pion-nucleon σ-term can be evaluated via the Feynman - Hellmann theorem [11, 12] as:

$$\sigma_{\pi N} = \hat{m}^0 (\frac{\partial M_N}{\partial m_u^0} + \frac{\partial M_N}{\partial m_d^0}), \tag{3}$$

where $\hat{m}^0$ stands for the average value of the current u and d quarks, $\hat{m}^0 = \frac{1}{2}(m_u^0 + m_d^0)$. Within the constituent quark model with chiral dynamics [5, 6], the nucleon mass consists of four terms:

$$M_N = \sum_{k=1}^{3} m_k + < N|H_{kin}|N > + < N|H_{conf}|N > + < N|H_\chi|N >, \tag{4}$$

where the second, third and fourth terms are contributions from the kinetic energy of the constituent quarks, confining interaction, and the GBE interaction between constituent quarks, respectively. Thus in order to evaluate (3) one needs an explicit dependence of each term in (4) on the current quark masses.

The constituent mass m_i includes the current quark mass value m_i^0 as well as a dynamical part m_i^D:

$$m_i = m_i^0 + m_i^D. \tag{5}$$

The latter appears from the spontaneous chiral symmetry breaking. In the chiral limit, $m_i^0 = 0$, the constituent quark mass is determined by the quark condensates $< \bar{q}q >$, which, in turn, are defined as the closed quark loops. Thus the dynamical part in (5) is in principle dependent on the full mass m_i and equation (5) becomes a gap (Schwinger-Dyson) equation. Obviously, no solution of this equation that takes into account full gluodynamics is presently available. Nevertheless, near the chiral limit, $m_u^0 = m_d^0 = 0$, the dynamical part in (5) is weakly dependent on the current quark masses m_u^0 and m_d^0. This feature is well seen from the solution of the gap

equation in the Nambu and Jona-Lasinio model well beyond the critical value of the coupling constant [4]. Thus, for a rough estimate one can use (near the chiral limit)

$$\frac{\partial m_i}{\partial m_j^0} \simeq \delta_{ij}, \tag{6}$$

where $i, j = u$ or d.

The kinetic term in (4) exibits m^{-1} dependence on the constituent quark mass, and thus

$$< N|\frac{\partial H_{kin}}{\partial m_u^0} + \frac{\partial H_{kin}}{\partial m_d^0}|N >= -\frac{1}{m} < N|H_{kin}|N > . \tag{7}$$

Here and in what follows we assume for simplicity equal masses for the constituent u and d quarks, $m_u = m_d = m$.

Assuming that the confining interaction is determined by the gluodynamics, one concludes that the confining term in (4) does not contribute to $\sigma_{\pi N}$. This also follows from the fact that the effective confining interaction between the constituent quarks does not depend on their masses.

The GBE interaction between the constituent quarks is proportional to m^{-2} [5, 6], and thus

$$< N|\frac{\partial H_\chi}{\partial m_u^0} + \frac{\partial H_\chi}{\partial m_d^0}|N >= -\frac{2}{m} < N|H_\chi|N > . \tag{8}$$

The repulsive contribution to the nucleon mass of the Yukawa tail of the quark-quark interaction, $\sim \mu^2 \exp{(-\mu r)}/r$, where μ is the meson mass, is very small [6], and hence the dependence of H_χ on m_u^0 and m_d^0 via meson mass μ in the Yukawa tail is not important and is neglected in (8). The crucial attractive contribution to the nucleon mass from the GBE comes from its short-range part which is μ-independent and has opposite sign relative to the Yukawa tail. It is this opposite sign which is the key to

the explanation of the baryon spectrum [5]. In the chiral limit the long-range Yukawa tail vanishes, while the short-range part of GBE remains intact.

Using in what follows the average value for the light quark masses $\hat{m}^0 = 7$ MeV [13] and a standard value $m = 340$ MeV for the constituent quark mass (which is suggested by the nucleon magnetic moments and which is used in the parametrization of the qq potential in [6]), the pion-nucleon sigma-term can now be estimated (3). For that we shall use the numerical values of $< N|H_{kin}|N >$ and $< N|H_\chi|N >$ developed in three-body Faddeev calculations in ref. [6]. With a parametrization of the GBE given therein, one has:

$$< N|H_{kin}|N >= 844 \ MeV,$$

$$< N|H_\chi|N >= -1130 \ MeV,$$

$$< N|H_{conf}|N >= 204 \ MeV. \tag{9}$$

The sum of all these terms plus $3m = 1020$ MeV gives just the nucleon mass.

One then obtains:

$$\sigma_{\pi N} \simeq 3 \times 7 - \frac{844}{340} \times 7 + 2\frac{1130}{340} \times 7 = 50.1 \ MeV \tag{10}$$

This result is in good agreement with the empirical value (2) as well as with the recent lattice QCD calculation [14], where $\sigma_{\pi N} = 47 - 53$ MeV.

One cannot insist, however, that it is the value $\hat{m}^0 = 7$ MeV which is responsible for the result (10). It would be so if the assumption (6) were exact. In fact the numerical value of the $\sigma_{\pi N}$ is determined by the products $\hat{m}^0 \frac{\partial m_i}{\partial m_j^0}$, thus it is better

to say that the result (10) is achieved with $\hat{m}^0(\frac{\partial m_i}{\partial m_i^0} + \frac{\partial m_i}{\partial m_j^0}) = 7$ MeV. Hence, the smaller values for the current quark mass could also be compatible with the empirical value of $\sigma_{\pi N}$ provided that the dependence of m_i^D on m_j^0 in Eq. (5) is essential. This question could be answered only when we have achieved a better understanding of a microscopical nature of the constituent quark. This uncertainty does not affect however the main conclusion that is discussed below.

Usually the nucleon is considered as a system of three weakly interacting constituent quarks as $M_N \simeq 3m$. This is not an adequate view. It can be seen from the $\Delta - N$ mass splitting that a difference of the expectation values of the spin-spin forces between the quarks for N and Δ should be of order 300 MeV. It is clear that the contribution of the spin-spin interaction to the nucleon has to be much bigger than the difference above. The big binding effect from the GBE is compensated mostly by the large kinetic energy as well as by the confining interaction. Such a compensation is well seen in (9). However, there is no such a compensation between $< N|H_\chi|N >$ and $< N|H_{kin}|N >$ contributions in $\sigma_{\pi N}$ in Eq. (10) as the weight factor of $< N|H_\chi|N >$ is twice bigger than the corresponding weight factor of the kinetic energy contribution as it is seen in Eqs. (7) and (8). Thus it is the big absolute value of $< N|H_\chi|N >$ which is crucial for the explanation of the $\sigma_{\pi N}$ within the constituent quark model.

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